INTERNATIONAL BACCALAUREATE

MATHEMATICS

Higher Level

Thursday 6 November 1997 (afternoon)

Paper 1

2 hours

This examination paper consists of 20 questions.

The maximum mark for each question is 4.

The maximum mark for this paper is 80.

This examination paper consists of 15 pages.

INSTRUCTIONS TO CANDIDATES

Write your candidate reference number in this box:

DO NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required/Essential:

IB Statistical Tables
Electronic calculator
Ruler and compasses

Allowed/Optional:

A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$\cos 2\theta=2\cos^2\theta-1=1-2\sin^2\theta=\cos^2\theta-\sin^2\theta$$

If $\tan\frac{\theta}{2}=t$ then $\sin\theta=\frac{2t}{1+t^2}$ and $\cos\theta=\frac{1-t^2}{1+t^2}$

Integration by parts:

$$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$$

Standard integrals:

$$\int\frac{dx}{x^2+a^2}=\frac{1}{a}\arctan\frac{x}{a}+c$$

$$\int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+c \quad (|x|<a)$$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m=\frac{\Sigma f_i x_i}{\Sigma f_i},\qquad s=\sqrt{\frac{\Sigma f_i(x_i-m)^2}{\Sigma f_i}},\qquad i=1,2,\ldots,n$$

Binomial distribution:

$$p_x=\binom{n}{x}p^x(1-p)^{n-x},\quad x=0,1,2,\ldots,n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper (provided these are securely fastened to the cover sheet together with this examination paper).

1. Find all the possible values of k if $x = k$ is a solution of the equation

$$x^3 + kx^2 - x - k = 0 .$$

Working:

Answers:

2. Given an $m \times n$ matrix A and an $n \times p$ matrix B, find the orders of the matrices C and D such that

$$2A(-4B + C) = 3D .$$

Working:

Answers:

3. Independent events A and B are such that $p(A) = 0.2$ and $p(A \cup B) = 0.6$. Find $p(B)$.

Working:

Answer:

4. (a) Given that $\log_a b = \frac{\log_c b}{\log_c a}$, find the real numbers k and m such that $\log_9 x^3 = k \log_3 x$ and $\log_{27} 512 = m \log_3 8$.

(b) Find all values of x for which $\log_9 x^3 + \log_3 x^{\frac{1}{2}} = \log_{27} 512$.

Working:

Answers:

(a)

(b)

5. A curve is defined by $x = t^2 + \sin 2t$ and $y = t + \sin t$, where t is a real parameter.

(a) Find the gradient (slope) of the curve at the point where $t = 0$.

(b) Find the equation of the tangent to the curve at the point where $t = 0$.

Working:

Answers:

(a) ______________________

(b) ______________________

6. Given that $y = xe^{3x} + \ln x$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Working:

Answers:

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7. Let $z = 3 + ik$ and $\omega = k + 7i$, where $k \in \mathbb{R}$ and $i = \sqrt{-1}$.

(a) Express $\dfrac{z}{\omega}$ in the form $a + ib$ where $a, b \in \mathbb{R}$.

(b) For what values of k is $\dfrac{z}{\omega}$ a real number?

Working:

Answers:

(a) ______________________

(b) ______________________

8. Consider the letters of the word MATHEMATICS.

(a) How many different arrangements of all eleven letters can be found?

(b) How many of these arrangements begin and end with the letter A?

Working:

Answers:

(a) ______________________

(b) ______________________

9. Solve the equation $5 \sin x - 12 \cos x = 6.5$ for $0° \leqslant x \leqslant 360°$.

Working:

Answers:

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10. Find the positive integer value of n such that the coefficients of x^2 in the binomial expansions of $(1 + x)^{2n}$ and $(1 + 15x^2)^n$ are equal.

Working:

Answer:

11. Find $\int \frac{\mathrm{d}x}{\sqrt{6x - x^2 - 5}}$.

Working:

Answer:

12. (a) Find the inverse of the matrix

$$A = \begin{pmatrix} k & -1 \\ 1 & k \end{pmatrix}$$

where $k \in \mathbb{R}$.

(b) Hence, or otherwise, solve the simultaneous equations

$$kx - y = 2k$$

$$x + ky = 1 - k^2 .$$

Working:

Answers:

(a) ____________________

(b) ____________________

13. Find all values of x for which $\dfrac{1}{x-\sqrt{x}} \geqslant \dfrac{4}{15}$.

Working:

Answer:

14. Find the value(s) of a if the line $9x - y = 14$ is a tangent to the curve $y = x^3 - 3x + a$ at the point on the curve where $x = a$.

Working:

Answer:

15. A random variable, X, has probability density function, $f(x)$, where

$$f(x) = \begin{cases} \frac{1}{4}x, & 0 \leqslant x < 1 \\ \frac{1}{4}, & 1 \leqslant x < 3 \\ \frac{1}{12}(6-x), & 3 \leqslant x \leqslant 6 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the median value of X.

Working:

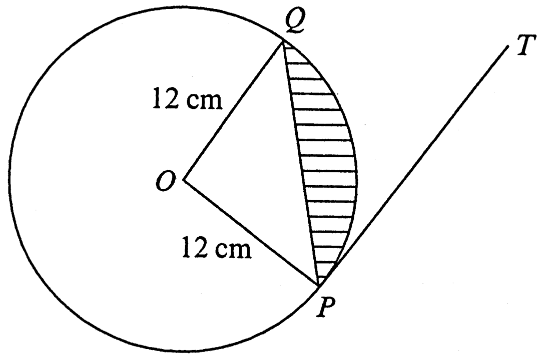
Answer:

16. The roots of the equation $x^2 + 5x + k = 0$, where k is a constant, are $x = \alpha$ and $x = \beta$. Find in terms of k, a quadratic equation whose roots are $x = \alpha^2$ and $x = \beta^2$.

Working:

Answer:

17. In the diagram, PT is a fixed tangent to a circle, centre O, radius 12 cm, and PQ is a chord of the circle.



If the angle TPQ is increasing at the rate of 3° per second, find the rate, in $cm^2\,s^{-1}$, at which the area of the shaded region is changing when the angle TPQ is 30°.

Working:

Answer: ______________________

18. An alarm clock is used to wake a student for school. The probability that the alarm rings is $\frac{4}{5}$. If the alarm rings, there is a probability of $\frac{7}{8}$ that the student arrives at school on time; but, if the alarm does not ring, the probability that the student arrives at school on time is $\frac{1}{10}$.

Find

(a) the probability that the student arrives at school on time on a given day ;

(b) the probability that, on a randomly chosen morning on which the student is late for school, the alarm did not ring.

Working:

Answers:

(a) ____________________

(b) ____________________

19. Find the locus of a point $P(x\,,y)$ in the complex plane, defined by the equation

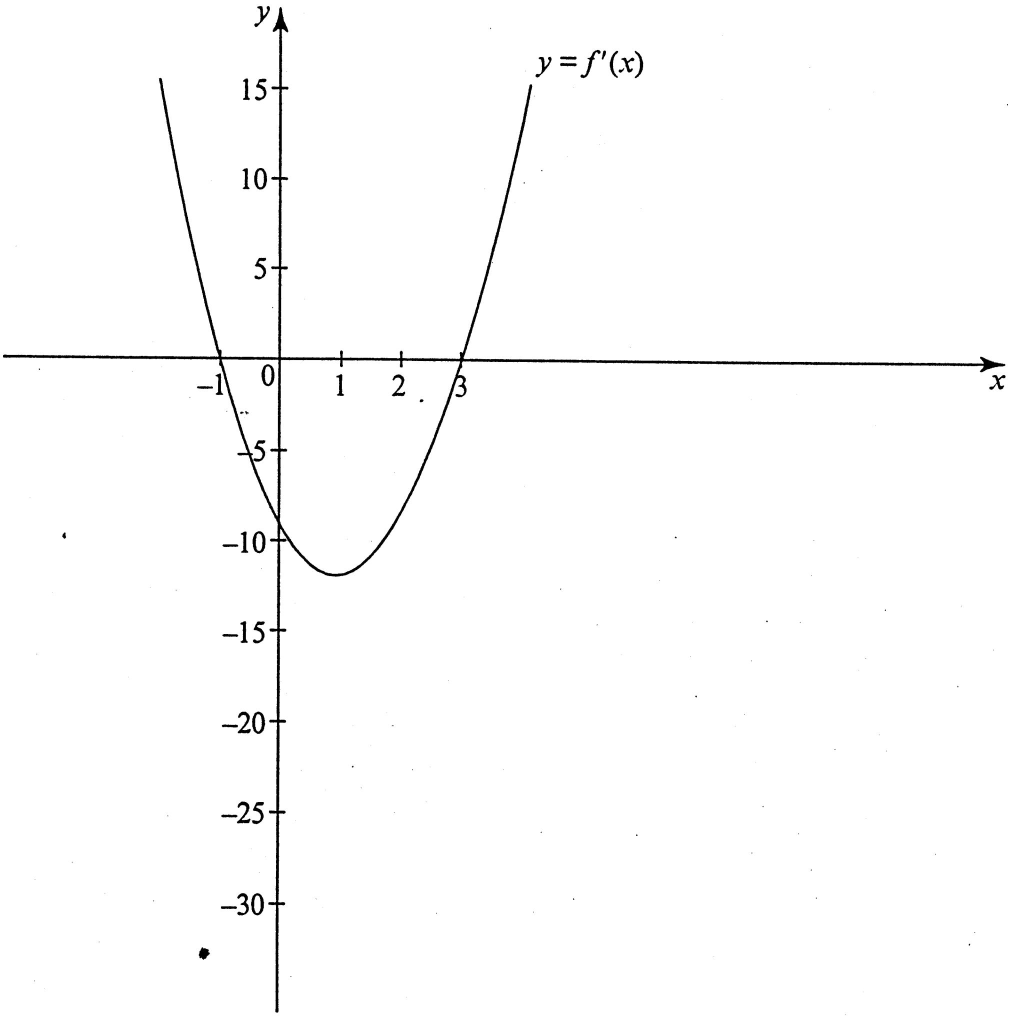
$$|1 - \mathrm{i}z| = |z + 1|,$$

where $z = x + \mathrm{i}y$ and $\mathrm{i} = \sqrt{-1}$.

Working:

Answer:

20. The graph of $f'(x)$, the derivative of the function $f(x)$, is shown. Given that $f(-1) = 5$, $f(0) = 0$, and $f(3) = -27$, sketch the graph of $y = f(x)$ on the same set of axes.



Working: